

Code No: 153CF

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, February - 2024

DISCRETE MATHEMATICS

(Common to CSE(AI&amp;ML), CSE(DS), CSE(IOT), AI&amp;DS, AI&amp;ML, CSD)

Time: 3 Hours

Max. Marks: 75

**Note:** i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

**PART - A****(25 Marks)**

- 1.a) Explain logical NAND. [2]
- b) If  $p \rightarrow q$  is true and  $r \rightarrow \sim q$  is true then prove that  $p \rightarrow \sim r$  is true. [3]
- c) State reflexive closure. [2]
- d) Let  $A = \{a, b, c, d\}$  and  $B = \{1, 2, 3\}$ . Let  $R = \{(a, 1), (a, 2), (b, 1), (c, 2), (d, 1)\}$ . Find the relation matrix. [3]
- e) State recurrence function. [2]
- f) Show that any positive integer  $n$  greater than or equal to 2 is either a prime or a product of primes. [3]
- g) If A and B are independent and  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{1}{4}$  and find  $P(A \cap B)$ . [2]
- h) Solve the recurrence relation  $a_{n+1} = 4a_n$  for  $n \geq 0$  give that  $a_0 = 3$ . [3]
- i) What is simple graph? Is it possible to draw a simple graph with 4 vertices and 7 edges? Justify your answer. [2]
- j) How many spanning trees does  $K_4$  have? [3]

**PART - B****(50 Marks)**

2. Construct truth table for the following  $(p \leftrightarrow \sim r) \wedge [(r \rightarrow q) \vee \sim q]$ . [10]

**OR**

3. Obtain the Principal disjunctive normal form of  $(p \rightarrow q) \wedge (q \leftrightarrow r)$ . [10]

4. Let  $A = \{1, 2, 3, 4\}$  and  
Let  $R_1 = \{(1, 1), (1, 2), (2, 3), (2, 4), (3, 4), (4, 1), (4, 2)\}$  and  
 $R_2 = \{(3, 1), (4, 4), (2, 3), (2, 4), (1, 1), (1, 4)\}$ , Verify (a)  $M_{R_1 R_2} = M_{R_1} \cdot M_{R_2}$   
(b)  $M_{R_1^c} = \text{Transpose of } M_{R_1}$  (c)  $M_{(R_1 R_2)^c} = M_{R_2^c} \cdot M_{R_1^c}$ . [4+3+3]

**OR**

5. The function  $f: R \rightarrow R$  be defined  $f(x) = 2x - 3$ , and  $f: R \rightarrow R$  by  $g(x) = x^2 + 5$  then, find the value of  $(fog)^{-1}(x), (gof)^{-1}(x)$ . [10]

6. Prove that mathematical induction  $n^3 - 4n^2$  is divisible by 3 for all  $n \geq 2$ . [10]

OR

7. Prove by the principle of Mathematical induction that

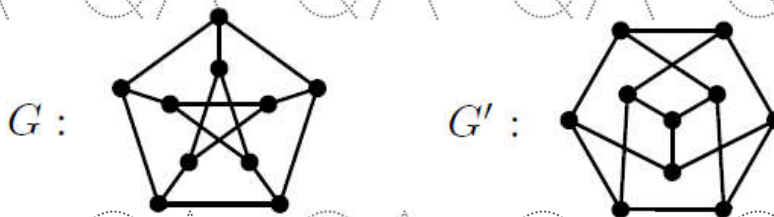
$$P(n): 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}. \quad [10]$$

8. A bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the red ball drawn is from bag B. [10]

OR

9. Solve the recurrence relation  $a_{n+2} - 10a_{n+1} + 21a_n = 3n^2 - 2, n \geq 0$ . [10]

10.a) By suitably lettering the vertices, prove that the following two graphs are isomorphic: Graph 1.



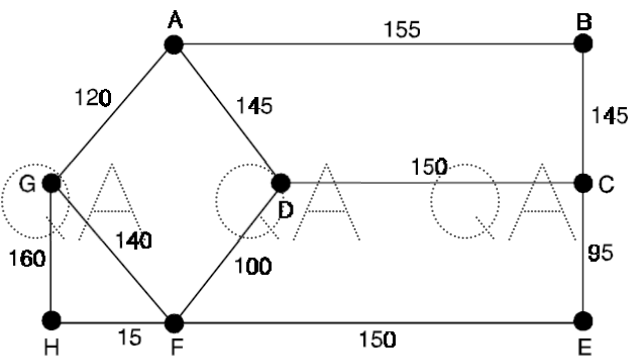
Graph 1

b) Characterize graphs which are both Eulerian and Hamiltonian. [5+5]

OR

11.a) Let  $T$  be a tree with  $\alpha$  vertices of degree 1 and  $\beta$  other vertices. Show that the sum of the degrees of the vertices of degree greater than 1 is  $\alpha + 2(\beta - 1)$ .

b) Use Kruskal's algorithm to find all least weight spanning trees for the following weighted graph. Graph 2 [5+5]



Graph 2